A Generalized Optimum Temperature Operations Criterion for a Deactivating Immobilized Enzyme Batch Reactor

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For a solid catalyzed single irreversible reaction $A \xrightarrow{k} B$ in a deactivating isothermal fixed-bed or batch reactor, the optimum temperature operations policy is to maintain a constant exit conversion for a fixed-bed reactor (Chou et al., 1967; Crowe, 1970; Crowe and Lee, 1971; Lee and Crowe, 1970*b*; Levenspiel and Sadana, 1978; Ogunye and Ray, 1968; Park and Levenspiel, 1976) and a constant effective rate coefficient for a batch reactor (Lee and Crowe, 1970*a*; Szepe, 1966; Szepe and Levenspiel, 1968) if the temperature is unconstrained. For the batch reactor this requires

$$ka = \text{constant} \qquad (1)$$

that is, raise the temperature of operations so as to just counter the drop in catalyst activity.

The optimum temperature operations policy for the isomerization of D glucose to D fructose catalyzed by glucose isomerase in solution in a batch reactor was determined by the formulation of a calculus of variations problem (Haas et al., 1974). However, the reaction was assumed to obey first-order reversible kinetics along with substrate independent enzyme deactivation. For the Langmuir-Hinshelwood type of Michaelis-Menten kinetics, neither Equation (1) nor the method of Haas et al. (1974) represent optimum temperature operations for deactivating immobilized enzyme catalyzed batch reaction systems. The purpose of this communication is to obtain a generalized optimum temperature operations criterion for immobilized enzyme catalyzed reactions exhibiting deactivation kinetics represented by

$$-\frac{dE}{dt} = k_d g_1(E) g_2(X_A, T) \qquad (2)$$

in an isothermal and diffusion free batch reactor. In fact, the development presented below applies equally well to soluble enzymes. The generalized form of deactivation kinetics, Equation (2), is apparently required to represent the past history (Szepe and Levenspiel, 1971) of some reactions, for example, when the substrate completely protects the enzyme from deactivation (Laidler and Bunting, 1973).

By optimum operations we mean that way of running the batch reactor which gives

highest $X_{A,\text{final}}$ for given E_{final} and t_{run}
highest E_{final} for given $X_{A,\text{final}}$ and t_{run}
shortest t_{run} for given $X_{A,\text{final}}$ and E_{final}

where $X_{A,\text{final}}$ and E_{final} are conversion of substrate and enzyme activity, respectively, at the end of the run t_{run}. In general, the allowable operating temperature will be subject to an upper and lower constraint T^* and T_*, respectively.

THEORY

$$\frac{dX_A}{dt} = k(T)F(X_A, T)E \quad \text{with} \quad k = k_o e^{-E_r/RT} \qquad (3)$$

$$\frac{-dE}{dt} = k_d g_1(E) g_2(X_A, T) \quad \text{with} \quad k_d = k_{do} e^{-E_d/RT} \qquad (2)$$

Let $F(X_A, T) = (1 - X_A)/(K_m' + 1 - X_A)$ for simple Michaelis-Menten kinetics, where $1 - X_A = Y_A$ is the normalized substrate concentration, S_A/S_{Ao}, $K_m' = K_{mo}' \exp(-E_m/RT)$, and T is temperature. Other types of enzyme catalyzed reactions such as competitive inhibition, noncompetitive inhibition, uncompetitive inhibition, and ordered bireactant are also reducible to the Michaelis-Menten type of form where F in each case is different (Segel, 1975). Other enzymatic reactions nonreducible to the Michaelis-Menten type of form may also be treated by the method given below.

The optimum temperature operations criterion for Equations (2) and (3) may be obtained from the Pontryagin maximum principle (Pontryagin et al., 1962). In the

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form of the Pontryagin maximum principle, the adjoint variables are written as

$$\frac{d\lambda_1}{dt} = -\lambda_1 E k \left(\frac{\partial F}{\partial X_A}\right)_T + \lambda_2 k_d g_1 \left(\frac{\partial g_2}{\partial X_A}\right)_T \qquad (4)$$

$$\frac{d\lambda_2}{dt} = -\lambda_1 k F + \lambda_2 k_d g_2 \frac{\partial g_1}{\partial E} \qquad (5)$$

The Hamiltonian is

$$H = \lambda_1 E k F - \lambda_2 k_d g_1 g_2 \qquad (6)$$

The unconstrained arc P is found by setting

$$\frac{\partial H}{\partial k_d} = 0 = \lambda_1 p k k_d^{-1} E F + \lambda_1 k E \frac{\partial F}{\partial k_d} - \lambda_2 g_1 g_2 - \lambda_2 k_d g_1 \frac{\partial g_2}{\partial k_d} \qquad (7)$$

where $k \propto k_d^p$.

Thus, from Equations (6) and (7), the Hamiltonian along the unconstrained arc P is given by

$$H = \lambda_1 E k F \left[1 - \frac{\left(p + \frac{k_d}{F}\frac{\partial F}{\partial k_d}\right)}{\left(1 + \frac{k_d}{g_2}\frac{\partial g_2}{\partial k_d}\right)} \right] \qquad (8)$$

Along the arc, P the Hamiltonian H is constant. Therefore, from Equation (8)

$$\frac{1}{\lambda_1}\frac{d\lambda_1}{dt} + \frac{\dfrac{d}{dt}\left\{ E k F \left[1 - \dfrac{\left(p + \frac{k_d}{F}\frac{\partial F}{\partial k_d}\right)}{\left(1 + \frac{k_d}{g_2}\frac{\partial g_2}{\partial k_d}\right)} \right] \right\}}{E k F \left[1 - \dfrac{\left(p + \frac{k_d}{F}\frac{\partial F}{\partial k_d}\right)}{\left(1 + \frac{k_d}{g_2}\frac{\partial g_2}{\partial k_d}\right)} \right]} = 0 \qquad (9)$$

From Equation (4)

$$\frac{1}{\lambda_1}\frac{d\lambda_1}{dt} + E k \left(\frac{\partial F}{\partial X_A}\right)_T = \frac{\lambda_2}{\lambda_1} k_d g_1 \left(\frac{\partial g_2}{\partial X_A}\right)_T \qquad (10a)$$

and from Equations (3), (7), and (10a)

$$\frac{1}{\lambda_1}\frac{d\lambda_1}{dt} + \frac{1}{F}\left(\frac{\partial F}{\partial X_A}\right)_T \frac{dX_A}{dt} = \frac{1}{g_2}\left(\frac{\partial g_2}{\partial X_A}\right)_T \frac{\left(p + \frac{k_d}{F}\frac{\partial F}{\partial k_d}\right)}{\left(1 + \frac{k_d}{g_2}\frac{\partial g_2}{\partial k_d}\right)} \frac{dX_A}{dt} \qquad (10b)$$

Therefore, from Equations (9) and (10b)

$$\frac{\dfrac{d}{dt}\left\{ E k F \left[1 - \dfrac{\left(p + \frac{k_d}{F}\frac{\partial F}{\partial k_d}\right)}{\left(1 + \frac{k_d}{g_2}\frac{\partial g_2}{\partial k_d}\right)} \right] \right\}}{E k F \left[1 - \dfrac{\left(p + \frac{k_d}{F}\frac{\partial F}{\partial k_d}\right)}{\left(1 + \frac{k_d}{g_2}\frac{\partial g_2}{\partial k_d}\right)} \right]} = \frac{1}{F}\left(\frac{\partial F}{\partial X_A}\right)_T \frac{dX_A}{dt} - \frac{1}{g_2}\left(\frac{\partial g_2}{\partial X_A}\right)_T \frac{\left(p + \frac{k_d}{F}\frac{\partial F}{\partial k_d}\right)}{\left(1 + \frac{k_d}{g_2}\frac{\partial g_2}{\partial k_d}\right)} \frac{dX_A}{dt} \qquad (11)$$

Equation (11) on integration gives

$$E k F \left[1 - \frac{\left(p + \frac{k_d}{F}\frac{\partial F}{\partial k_d}\right)}{\left(1 + \frac{k_d}{g_2}\frac{\partial g_2}{\partial k_d}\right)} \right] \exp\left[\int_0^{X_A} \left(-\frac{1}{F}\frac{\partial F}{\partial X_A}\right)_T dx_A \right] \exp\left[\int_0^{X_A} \left(\frac{1}{g_2}\frac{\partial g_2}{\partial X_A}\right)_T \frac{\left(p + \frac{k_d}{F}\frac{\partial F}{\partial k_d}\right)}{\left(1 + \frac{k_d}{g_2}\frac{\partial g_2}{\partial k_d}\right)} dX_A \right] = \text{constant} \qquad (12)$$

Equation (12) is a generalized optimum temperature operations criterion for a deactivating immobilized enzyme catalyzed reaction in an isothermal and diffusion free batch reactor and may be utilized to obtain the specific optimum temperature operations criterion for the different reaction and deactivation cases given below. Note that $g_1 = g_1(E)$ does not appear in the generalized optimum temperature operations criterion.

Case I: $g_1 = g_1(E)$, $F = F(X_A, T)$, and $g_2 = g_2(X_A, T)$

When the substrate completely protects the immobilized enzyme from deactivation (Laidler and Bunting, 1975), the reaction and deactivation expressions are given by

$$\frac{dX_A}{dt} = k E F \qquad (3)$$

$$-\frac{dE}{dt} = \frac{k_d E K_m'}{K_m' + 1 - X_A} = k_d g_1(E) g_2(X_A, T) \qquad (13)$$

for simple Michaelis-Menten kinetics where $F = (1 - X_A)/(K_m' + 1 - X_A)$, $g_2 = K_m'/(K_m' + 1 - X_A)$, and $g_1(E) = E$.

Substitute for F and g_2 in Equation (12). Then, Equation (12) on simplification gives

$$k E \frac{(1 - X_A)}{K_m' + 1 - X_A} \left[\frac{K_m'(1 - (p - q)) + (1 + q - p)(1 - X_A)}{K_m' + (1 + q)(1 - X_A)} \right] \exp\left[\int_0^{X_A} \frac{K_m'}{(1 - X_A)(K_m' + 1 - X_A)} dX_A \right] \exp\left[\int_0^{X_A} \frac{1}{K_m' + 1 - X_A} \frac{(p - q)K_m' + p(1 - X_A)}{K_m' + (1 + q)(1 - X_A)} dX_A \right] = \text{constant} \qquad (14)$$

as the optimum temperature operations criterion, where $K_m' \propto k_d^q$ and q is a constant.

Case II: $g_1 = g_1(E)$, $F = F(X_A, T)$, and $g_2 = g_2(X_A)$

The generalized optimum temperature operations criterion, Equation (12), now reduces to

$$kEF\left[1-\left(p+\frac{k_d}{F}\frac{\partial F}{\partial k_d}\right)\right] \exp\left[\int_0^{X_A}\left(-\frac{1}{F}\frac{\partial F}{\partial X_A}\right)_T dX_A\right] \exp\left[\int_0^{X_A}\frac{1}{g_2}\frac{dg_2}{dX_A}\left(p+\frac{k_d}{F}\frac{\partial F}{\partial k_d}\right)dX_A\right] = \text{constant} \tag{15}$$

When the substrate protects the immobilized enzyme activity to a certain extent (Haldane, 1930; Lamba and Dudukovic, 1974; O'Neill, 1972; Scrutton and Utter, 1965; Staub and Denes, 1967), the reaction and deactivation expressions are given by

$$\frac{dX_A}{dt} = kEF \tag{3}$$

and

$$-\frac{dE}{dt} = k_dE\frac{1}{1-X_A} = k_dg_1(E)g_2(X_A) \tag{16}$$

for simple Michaelis-Menten kinetics, where $F = (1 - X_A)/(K_m' + 1 - X_A)$, $g_1(E) = E$, and $g_2 = 1/(1 - X_A)$.

Substitute for F and g_2 in Equation (15). Then, Equation (15) on simplification gives

$$kE(1-X_A)\left[\frac{(1-p+q)K_m' + (1-X_A)(1-p)}{(K_m'+1-X_A)^2}\right] \exp\left[\int_0^{X_A}\frac{K_m'}{(1-X_A)(K_m'+1-X_A)}dX_A\right] \exp\left[\int_0^{X_A}\frac{1}{1-X_A}\frac{(p-q)K_m'+p(1-X_A)}{K_m'+1-X_A}dX_A\right] = \text{constant} \tag{17}$$

as the optimum temperature operations criterion.

Case III: $g_1 = g_1(E)$, $F = F(X_A, T)$, and $g_2 = 1$

The generalized optimum temperature operations criterion, Equation (12), now reduces to

$$kEF\left[1-\left(p+\frac{k_d}{F}\frac{\partial F}{\partial k_d}\right)\right] \exp\left[\int_0^{X_A}\left(-\frac{1}{F}\frac{\partial F}{\partial X_A}\right)_T dX_A\right] = \text{constant} \tag{18}$$

For competitive inhibition (Segel, 1975) and for first-order substrate independent immobilized enzyme deactivation, the reaction and deactivation rate expressions are given by

$$\frac{dX_A}{dt} = kEF \tag{3}$$

and

$$-\frac{dE}{dt} = k_dE = k_dg_1(E) \tag{19}$$

where now $F = (1 - X_A)/[K_m'(1 + (I/K_I)) + 1 - X_A]$, I is inhibitor concentration, K_I is the equilibrium rate constant for the inhibition reaction, and $g_1(E) = E$. Substitute for F in Equation (18). Then, Equation (18) on simplification gives

$$\frac{kE(1-X_A)}{\{K_m'[1+(I/K_I)]+1-X_A\}^2}\left[\{K_m'[1+(I/K_I)] + 1 - X_A\}(1-p) + qK_m' + (q-i)\frac{K_m'}{K_I}I\right] \exp\left[\int_0^{X_A}\frac{K_m'[1+(I/K_I)]}{(1-X_A)\{K_m'[1+(I/K_I)]+1-X_A\}}dX_A\right] = \text{constant} \tag{20}$$

as the optimum temperature operations criterion, where $K_I \propto k_d^i$ and i is a constant.

Case IV: $g_1 = g_1(E)$, $F = F(X_A)$, and $g_2 = g_2(X_A)$

The generalized optimum temperature operations criterion, Equation (12), now reduces to

$$kEF(1-p)\exp\left[\int_0^{X_A}-\frac{1}{F}\frac{dF}{dX_A}dX_A\right] \exp\left[\int_0^{X_A}\frac{p}{g_2}\frac{dg_2}{dX_A}dX_A\right] = \text{constant} \tag{21}$$

Equation (21) on simplification yields

$$kE[g_2(X_A)]^p = \text{constant} \tag{22}$$

which was shown by Lee and Crowe (1970*a*) to be the optimum temperature operations criterion for batch reactors with decaying catalyst, where F is a function of X_A only, and deactivation is dependent on the extent of reaction.

Case V: $g_1 = g_1(E)$, $F = F(X_A)$, $g_2 = 1$

The generalized optimum temperature operations criterion, Equation (12), now reduces to

$$kEF(1-p)\exp\left[\int_0^{X_A}-\frac{1}{F}\frac{dF}{dX_A}dX_A\right] = \text{constant} \tag{23}$$

Equation (23) on simplification gives

$$kE = \text{constant} \tag{24}$$

which was shown by Szepe and Levenspiel (1968) and Crowe (1970) to be the optimum temperature operations criterion for batch reactors with decaying catalyst, where F is a function of X_A only, and deactivation is independent of the extent of reaction.

CONCLUSIONS

A generalized optimum temperature operations criterion is developed for deactivating immobilized enzyme catalyzed reactions, where the expression for deactivation is of a general enough nature to represent the past history of the reaction in an isothermal and diffusion free batch reactor. The treatment applies equally well to soluble enzyme batch reactors. The development should, in general, be applicable to different types of enzymatic reactions which may or may not be reducible to the Michaelis-Menten type of form.

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NOTATION

a = activity of catalyst

E = activity (amount) of immobilized enzyme, g
E_r = activation energy for reaction, k cal/g mole
E_m = activation energy for Michaelis-Menten constant, k cal/g mole
E_d = activation energy for deactivation, k cal/g mole
F = generalized form of the immobilized enzyme catalyzed reaction expression
g_1, g_2 = deactivation functions
i = constant
I = inhibitor concentration, g/l
k = reaction velocity constant, $(s)^{-1}$ (other units dependent on step)
k_d = deactivation rate constant, $(s)^{-1}$ (other units dependent on step)
K_I = equilibrium rate constant for inhibitor reaction
K_m' = normalized Michaelis-Menten constant, K_m/S_{Ao}
p, q = constants
R = gas constant, cal $(\text{g mole} - {}^\circ\text{K})^{-1}$
S_A = substrate concentration, g/l
S_{Ao} = substrate concentration at $t = 0$, g/l
t = time, s
T = temperature, °K
X_A = fractional conversion of substrate A
Y_A = fraction of unconverted substrate A, S_A/S_{Ao}

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Effectiveness Factors in Trickle-Bed Reactors

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Downflow of gas and liquid over a bed of catalyst particles can result in partial coverage, or wetting, of the particles with flowing liquid at low liquid rates (Satterfield, 1975). When this occurs, the concentration around the outer surface of a spherical particle is not uniform, nor is the concentration field within the particle symmetrical with respect to radial position. In this situation, the usual expressions for the effectiveness factor are not applicable. What is needed for design is an overall effectiveness factor η_o based upon bulk stream concentrations, which accounts for the partial liquid coverage. Two approximate solutions are presented in this note; one is an explicit equation for η_o, and the other is a two-dimensional representation of the catalyst particle requiring numerical solution.

The overall effectiveness factor will include interphase transport effects, and these depend upon the type of reaction. When the limiting reactant is in the gas, but at least slightly soluble in the liquid, mass transfer from gas to liquid and liquid to particle must occur in order for reactant to reach the catalyst from the liquid covered surface. Only mass transfer from gas to particle is necessary for reactant to reach the catalyst on the gas covered part of the particle. An example is the removal of a pollutant from a liquid by catalytic oxidation with air in a trickle-bed reactor. Another possibility is that the limiting reactant is in the liquid phase. Now, reactant reaches the catalyst only through the liquid covered part of the particle, and only liquid to particle mass transfer

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